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THEORETICAL AND EXPERIMENTAL ACOUSTIC MODAL ANALYSIS OF REACTIVE FILTERS IN PIPING SYSTEM

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Abstract. *Reactive filters in piping system are a connection of expansion chambers and pipes. The principal feature is that the combination of sudden changes of these elements can act as a filter which can be applied to reduce noise at low frequencies. The reduction of the noise will depend on the eigenvalues of the system. In this paper the theoretical and experimental modal analyses of such reactive filters is investigated. An acoustical system composed of 2 chambers and 3 pipes connected in series is used as an example. Both a lumped acoustical model and a Finite Element model are used. The theoretical predictions are compared with experimental results obtained using a modal parameter extraction method that uses the measured Frequency Response Functions of the acoustic system. While, in the case of mechanical systems, analytical and experimental modal analyses are widely used, for acoustical systems this is certainly not true. The main purpose of the paper is to show the similarities between acoustic and mechanical experimental modal analyses.*

Key words: *Acoustic modal analysis, Reactive filters, Piping systems, Modal analysis.*

1 INTRODUCTION

In many applications of ducts and pipes, such as car exhaust systems and ventilation systems in buildings, the use of acoustical filters helps to reduce undesired noise at certain frequencies, in the form of either standing or propagating acoustical waves. The analytical and experimental modal analyses of these acoustical systems is a useful tool for their design and optimization.

While, in the case of mechanical systems, theoretical and experimental modal analyses (Ewins,1998) are widely used, for acoustical systems this is certainly not true. In the current literature on acoustic systems little attention is paid to defining excitation and response acoustic variables such that an experimental modal analysis is feasible. Frequently, the variable used to write the dynamic system equations is the velocity potential, which is not directly measurable and impossible to impose to an acoustic system. In other cases pressure is used as excitation and volume velocity as response, which is again not practical for experimental implementation.

Augusztinovicz and Sas (1996) have recently addressed this problem. They have proposed a formulation where volume acceleration is the input variable and pressure the response variable in the dynamic equations of the acoustical system. Pressure may be easily measured with microphones, while volume acceleration can be produced by calibrated sound sources such as loudspeakers in specially designed configurations. It must be pointed out that the problem of calibrated sound sources for acoustical modal analysis is still an open one, in the sense that there is no commercially available small calibrated sound source for acoustical modal analysis.

In this paper, the work of Augusztinovicz and Sas (1996) is complemented with an experimental acoustical modal analysis of a simple acoustic reactive filter. At low frequencies, the system may be treated as a lumped acoustical system with three "acoustical masses" (inertances) and two acoustical "springs" (compliances) (Kinsler *et al.*,1982). The impedances of the two basic acoustic components in a reactive filter - inertances and compliances - are reviewed. First, the usual formulation, where the excitations are pressures and the responses volume velocities, is presented. Then, the formulation that is more appropriate to an experimental modal analysis, where volume acceleration is the excitation and the pressures are the responses, is shown.

A Finite Element (FE) model of the system is obtained and the system matrices are solved to yield the natural frequencies and mode shapes. Linear triangular axisymmetric finite elements are used to solve the wave equation (Kwon & Bang, 1997) written using velocity potentials as variables.

The modal parameters of the lumped acoustical system obtained using the analytical lumped solution, the FE model solution, and experimental modal analysis are compared. The main purpose of the paper is to show the similarities between acoustic and mechanical experimental modal analyses.

2 LUMPED ACOUSTICAL SYSTEM IMPEDANCES

The acoustical impedance Z experienced by a sound wave acting on a surface of area S is the complex quotient of the acoustic pressure p divided by the volume velocity U at the surface.

Three types of acoustical impedance can be defined. The *specific acoustic impedance*, $z = \frac{p}{u}$ (pressure over particle velocity), used in three-dimensional acoustics, the *acoustic impedance*, $Z = \frac{p}{U}$ (pressure over volume velocity), generally used when investigating the sound propagation one-dimensional acoustic systems, and the *radiation impedance*, $Z_r = \frac{pS}{u} = zS$ (force over particle velocity), used when computing the coupling between sound waves and a driven load (Kinsler et al.,1982).

In order to obtain the impedance characteristics of the two lumped acoustic elements investigated here, namely pipes and expansion chambers (cavities), it is proposed to use two different formulations. In the first case, when investigating the pipe element, the excitation variable will be the pressure and the volume velocity the response (Fig. 1.a). In the second case, for the acoustical cavity, the excitation will be the volume velocity and the response the pressure (Fig. 1.b).

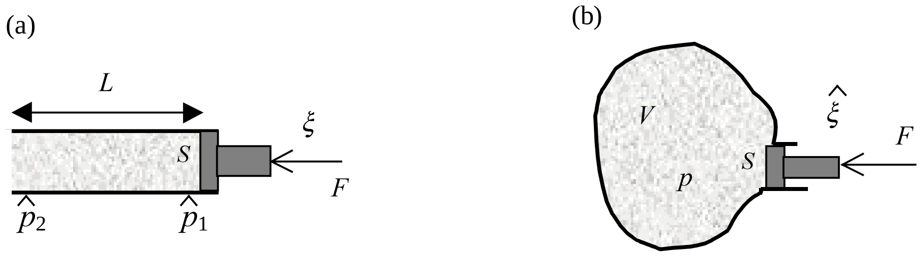

Figure 1. (a) For the pipe, the pressure is the excitation and the volume velocity the response. (b) For the cavity, the volume velocity is the excitation and the pressure the response.

2.1 Formulation with excitation by pressure (pipe)

The mechanical force F applied on the piston (Fig. 1.a) is equal to the pressure p times the cross-sectional area of the pipe, S. According to Newton's laws, the force is equal to the acceleration $\frac{d^2\xi}{dt^2}$ times the mass of the fluid contained in the volume SL .

$$F = (p_1 - p_2)S = \rho SL\frac{d^2\xi}{dt^2} \Rightarrow (p_1 - p_2) = p = \rho L\frac{d^2\xi}{dt^2} = \rho L\frac{du}{dt} \tag{1}$$

where F is the mechanical force applied, S the cross-sectional area of the pipe, p_1 and p_2 the pressures at locations 1 and 2, ξ the displacement, u the particle velocity, ρ the fluid density and L the length of the waveguide.

Using the equilibrium (Euler's) equation (Kinsler et al.,1982), the pressure in Eq. (1) in the frequency domain can be expressed as,

$$\hat{p} = i\omega\rho L\hat{u} \tag{2}$$

The volume velocity is the area multiplied by the particle velocity:

$$U = Su \Rightarrow \hat{U} = S\hat{u} \tag{3}$$

From Eqs. (2) and (3),

$$\hat{U} = \frac{1}{i\omega}\frac{S}{\rho L}\hat{p} = \frac{1}{i\omega M}\hat{p} \tag{4}$$

where $M = \frac{\rho L}{S}$ is called the acoustic inertance of the waveguide (Kinsler et al.,1982).

2.2 Formulation with excitation by volume velocity (cavity)

In the Fig. 1.b, a mechanical force F is applied on the piston, thus producing a displacement ξ . This will cause a change of the volume in the cavity $\Delta V = S\xi$ and, consequently, a condensation $\frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V} = \frac{S}{V}\xi$ of the fluid. The force F will be equal to the displacement ξ multiplied by the compressibility coefficient $\frac{\rho c^2 S}{V}$. On the other hand, the mechanical force is equal to the pressure times the surface area:

$$F = pS = \frac{\rho c^2 S^2}{V}\xi \tag{5}$$

where F is the mechanical force applied over the area S, p is the pressure in the cavity (assumed uniform in the cavity), ξ is the displacement of the particle over the surface S, and V is the volume of the cavity.

The constitutive equation of the fluid, within the linear acoustics framework, can be expressed in the frequency domain (Kinsler et al.,1982):

$$\hat{p} = \frac{\rho c^2 S}{V}\hat{\xi} \tag{6}$$

The volume velocity is the area multiplied by the speed of the particle:

$$U = S\frac{d\xi}{dt} \Rightarrow \hat{U} = i\omega S\hat{\xi} \tag{7}$$

Using Eqs. (6) and (7), we have

$$\hat{U} = i\omega\frac{V}{\rho c^2}\hat{p} = i\omega C\hat{p} \tag{8}$$

where $C = \frac{V}{\rho c^2}$, is the acoustic compliance of the cavity.

2.3 Acoustical system example

The simple reactive system example treated in this paper is shown in Fig. 2. It consists of three pipes, which can be modeled as three acoustical inertances, and two expansion chambers (cavities) which are modeled as acoustical compliances. The geometry of the system is given in Fig. 2, where dimensions are given in millimeters. The experiments were carried out at room temperature of approximately 20°C. Thus, the air density will be taken as 1.21 kg/m^3 and the sound velocity 343 m/s.

Given the physical dimensions of the system, it is possible to calculate the acoustical impedances in terms of acoustical inertances and acoustical compliances.

A correction of pipe lengths to take into account the radiation impedance at the pipe ends was performed according to (Kinsler et al.,1982): $L_1' = L_5' = L_1 + 1.5(d_1/2) = 122.2$ [mm] (inner end flanged, outer end unflanged), $L_3' = L_3 + 1.7(d_3/2) = 119.9$ [mm] (both ends flanged).

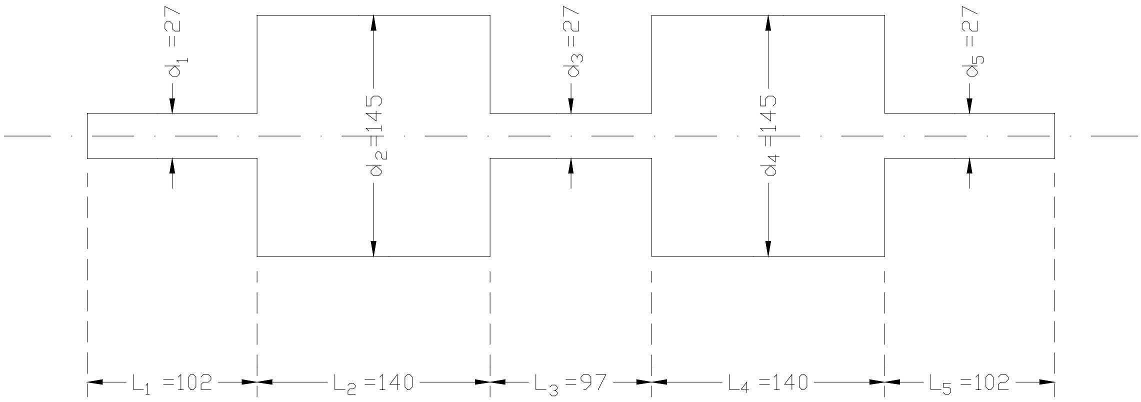

Figure 2. Acoustical system used as an example (dimensions in mm).

The acoustic inertances are $M_1 = M_3 = \rho L_1 \Big/ \left(\pi (d_1/2)^2\right) = 258.355$ [kg/m], $M_2 = \rho L_3 \Big/ \left(\pi (d_3/2)^2\right) = 253.494$ [kg/m] and the acoustic compliances are $C_1 = C_2 = \pi L_2 (d_2/2)^2 \big/ \rho c^2 = 1.624 \times 10^{-8}$ [$m^4 s^2$/kg].

As mentioned before, the dynamic acoustical system can be represented in two different ways: In the first, the pressure is used as the excitation variable and the volume velocity as the response. In the second case, the excitation is realized by a volume velocity and the pressure is the response. The equilibrium of pressures can be applied in the first situation to obtain the dynamical equations in the frequency domain.

$$i\omega M_1 \hat{U}_1 + \frac{1}{i\omega}\frac{1}{C_1}\hat{U}_1 - \frac{1}{i\omega}\frac{1}{C_1}\hat{U}_2 = \hat{p}_1 \tag{9}$$

$$i\omega M_2 \hat{U}_2 - \frac{1}{i\omega}\frac{1}{C_1}\hat{U}_1 + \frac{1}{i\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\hat{U}_2 - \frac{1}{i\omega}\frac{1}{C_2}\hat{U}_3 = \hat{p}_2 \tag{10}$$

$$i\omega M_3 \hat{U}_3 - \frac{1}{i\omega}\frac{1}{C_2}\hat{U}_2 + \frac{1}{i\omega}\frac{1}{C_2}\hat{U}_3 = \hat{p}_3 \tag{11}$$

where $\hat{U}_1$, $\hat{U}_2$ and $\hat{U}_3$ are the particle velocities in the pipes, and $\hat{p}_1$, $\hat{p}_2$ and $\hat{p}_3$ are the excitation pressures at the left end of each pipe. Expressing Eqs. (9), (10) and (11) in matrix form.

This formulation is not adequate for experimental validation, as volume velocities are very awkward to measure, and there are no general purpose volume velocity acoustical transducers.

$$\left\{ i\omega \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} + \frac{1}{i\omega} \begin{bmatrix} \frac{1}{C_1} & -\frac{1}{C_1} & 0 \\ -\frac{1}{C_1} & \left(\frac{1}{C_1}+\frac{1}{C_2}\right) & -\frac{1}{C_2} \\ 0 & -\frac{1}{C_2} & \frac{1}{C_2} \end{bmatrix} \right\} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} = \begin{Bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{Bmatrix} \tag{12}$$

In the second situation, using the conservation of mass in the acoustical cavities, the dynamical equations in the frequency domain can be written as:

$$i\omega C_1 \hat{p}_1 + \frac{1}{i\omega}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\hat{p}_1 - \frac{1}{i\omega}\frac{1}{M_2}\hat{p}_2 = \hat{U}_1 \tag{13}$$

$$i\omega C_2 \hat{p}_2 - \frac{1}{i\omega}\frac{1}{M_2}\hat{p}_1 + \frac{1}{i\omega}\left(\frac{1}{M_2}+\frac{1}{M_3}\right)\hat{p}_2 = \hat{U}_2 \tag{14}$$

where $\hat{U}_1$ and $\hat{U}_2$ are excitation volume velocities in the cavities and $\hat{p}_1$ and $\hat{p}_2$ are the pressure responses. Equations (13) and (14) can be expressed in matrix form as,

$$\left\{ i\omega \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} + \frac{1}{i\omega} \begin{bmatrix} \frac{1}{M_1}+\frac{1}{M_2} & -\frac{1}{M_2} \\ -\frac{1}{M_2} & \frac{1}{M_2}+\frac{1}{M_3} \end{bmatrix} \right\} \begin{Bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{Bmatrix} = \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{Bmatrix} \tag{15}$$

multiplying Eq. (15) by $i\omega$ and reordering,

$$\left\{ \begin{bmatrix} \frac{1}{M_1}+\frac{1}{M_2} & -\frac{1}{M_2} \\ -\frac{1}{M_2} & \frac{1}{M_2}+\frac{1}{M_3} \end{bmatrix} - \omega^2 \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \right\} \begin{Bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{Bmatrix} = \begin{Bmatrix} i\omega\hat{U}_1 \\ i\omega\hat{U}_2 \end{Bmatrix} \tag{16}$$

where the terms $i\omega\ \hat{U}_1$ and $i\omega\ \hat{U}_2$ are volume accelerations. Finally, substituting the acoustical parameters of the system in Fig. 2 in Eq. (16) yields,

$$\left\{ \begin{bmatrix} 7.80\times10^{-3} & -3.90\times10^{-3} \\ -3.90\times10^{-3} & 7.80\times10^{-3} \end{bmatrix} - \omega^2 \begin{bmatrix} 16.24\times10^{-9} & 0 \\ 0 & 16.24\times10^{-9} \end{bmatrix} \right\} \begin{Bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{Bmatrix} = \begin{Bmatrix} \hat{\dot{U}}_1 \\ \hat{\dot{U}}_2 \end{Bmatrix} \tag{17}$$

This formulation is more suitable for experimental validation, as the resulting Frequency Response Functions (FRFs) are given in terms of pressure per volume acceleration. As it will be shown in the experimental set-up section, it isn't difficult to implement a volume acceleration actuator.

3 FINITE ELEMENT MODEL

The wave equation can be written using velocity potentials as variables (Kwon & Bang, 1997),

$$\nabla^2 \varphi - \frac{1}{c^2}\ddot{\varphi} + d = 0 \tag{18}$$

where φ is the velocity potential ($\vec{u} = \vec{\nabla}\varphi$) and d the contribution of sound sources. Approximating the velocity potential distribution in the continuum using linear isoparametric triangular axisymmetric elements, the homogeneous dynamic system equations may be written as

$$[E]\{\ddot{\varphi}\} + [H]\{\varphi\} = \{0\} \tag{19}$$

where $[E]$ is usually called the compressibility matrix (associated with the potential energy) and $[H]$ the volumetric matrix (associated with the kinetic energy). From these two matrices the eigenvalues and eigenvectors can be easily obtained. A simple *ad hoc* routine was implemented in MATLAB® (The Mathworks, Inc.) to solve this problem. The mesh used is shown in Fig. 3. The model has a total of 252 degrees of freedom.

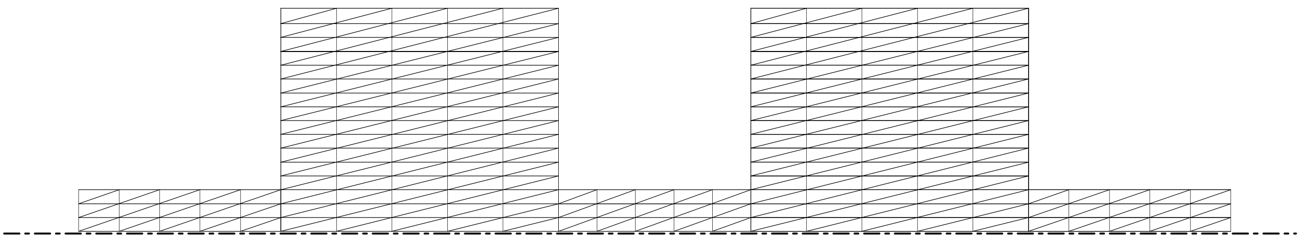

Figure 3. Acoustical Finite Element mesh.

4 EXPERIMENTAL SET-UP

The experimental set-up is shown schematically in Fig. 4. The geometry of the acoustical system was given in Fig. 2. The system is made of PVC. Both system terminations are open. Two microphones were used to measure the pressures in the cavities (electret microphones with nominal sensitivity of 25 mV/Pa). The calibrated volume acceleration actuator consisted of a PVC piston with a circular section of 25 mm diameter driven by an electrodynamic shaker with a piezoelectric accelerometer (nominal sensitivity 100 mV/g) mounted on it. The measured acceleration times the piston area is the volume acceleration. A thin rubber membrane was used to seal the gap between the piston and the circular hole opened in the side wall of the cavity. The gap was of approximately 1 mm.

5 RESULTS

Figure 5 shows a comparison of the measured and predicted FRFs and when cavity 1 is excited and pressures are measured in both cavities. One must keep in mind that the pressure is assumed constant in the lumped acoustical cavity element. The analytical FRF was predicted using Eq. (13). The agreement is very good except for the damping, which in this case is mainly due to the PVC wall of the cavities, and was not included in the analytical model.

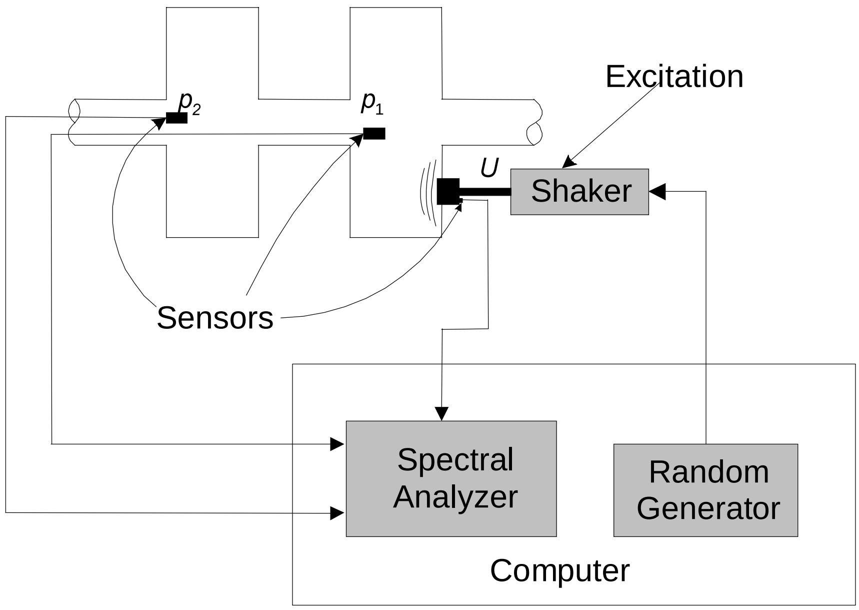

Figure 4. Experimental set-up

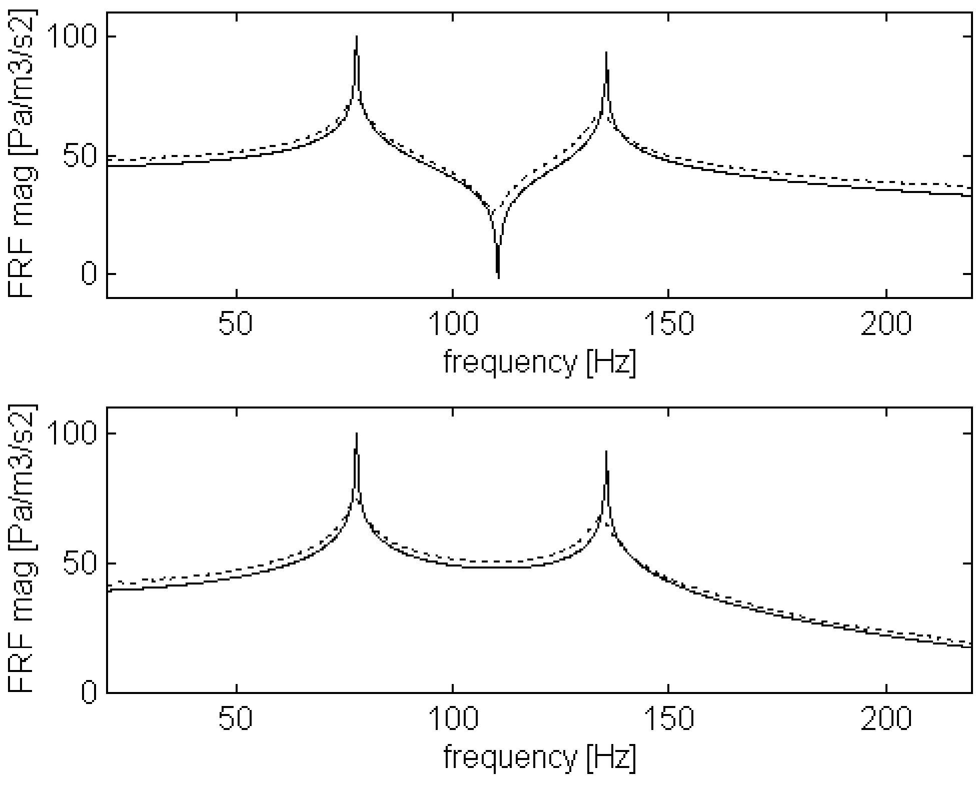

Figure 5. Comparison of analytical and experimental FRFs. — Analytical; --- experimental.

Table 1 shows a comparison of analytical and experimental modal parameters. In the case of the experimental results, the modal parameters were extracted from the measured FRFs using a frequency domain orthogonal polynomial method (Arruda *et al.*,1996). The agreement is good between both theoretical predictions and the experimental results, thus validating the experimental procedure for acoustical modal analysis.

While the lumped model predicts only the two first natural frequencies, the FE model can predict higher order modes where the pipes and cavities cannot be treated as lumped acoustical elements. The third natural frequency obtained experimentally was 1186 Hz and the FE prediction was 1201 Hz.

Table 1: Comparison of analytical and experimental modal parameters

Mode Number		Natural frequency [Hz]	Viscous damping factor	Mode shape DOF 1	Mode shape DOF 2
1	Analytical	77.70	0	1	1
	FEM	78.0	0	1	0.998
	Experimental	77.39	4.8×10^{-5}	1	0.991
2	Analytical	135.44	0	1	-1
	FEM	133.9	0	1	-0.995
	Experimental	134.17	6.3×10^{-5}	1	-0.954

6 CONCLUSIONS

An experimental acoustical modal analysis of a simple reactive filter was performed. Results agreed very well with the theoretical predictions made using both a lumped acoustical model and a Finite Element model. It was shown that it is possible to formulate the lumped acoustical model in such a way that volume accelerations are the inputs and pressures the outputs. This formulation is particularly suitable to generate FRFs which can be obtained experimentally. A simple piston system driven by an electrodynamic shaker was used to excite the acoustical system with a calibrated input. This procedure can be used in practical applications, so that true experimental modal analysis can be performed, instead of the current practice of measuring operational acoustical modes to validate model predictions.

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